JdS 2021: Propagation of epistemic uncertainties and global sensitivity analysis in seismic risk assessment

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Seismic hazard

Izmir seism picture (from newspaper *Le Dauphiné*, November 1st 2020)

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- Seisms have a huge social & human costs and are unfortunately random.
- Dramatic effect on mechanical structures.

Seismic hazard

Crucial need of seismic risk assesment for valuable industrial assets (e.g. nuclear power plant)

Nuclear power plant of Nogent sur Seine, France

Context: Seismic probabilistic risk assessment

Seismic probabilistic risk assessment (SPRA) is dedicated in estimating the safety of a mechanical structure subjected to seismic ground motions and consists in three main steps[1]:

- Seismic hazard probability distribution on a given site: $dh(a) = p(a)d\mu$
- Seismic fragility curve estimation $\Psi(a) = \mathbb{P}(Y > C|A = a)$. By definition the conditional probability of failure of the structure given a seismic intensity of level $A = a$.
- Our QoI: Final probability of failure:

$$\Upsilon = \int \Psi(a)dh(a)$$

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[1] Robert P. Kennedy. Risk based seismic design criteria. *Nuclear Engineering and Design*, 1999

Industrial motivations

- Uncertainties are divided in two groups:
  - *Aleatory*: Natural variability of a physical phenomenon.
  - *Epistemic*: Comes from the Greek word $\epsilon\pi\iota\sigma\tau\eta\mu\eta$ (knowledge). Uncertainties resulting from a lack of knowledge

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[2] Armen Der Kiureghian and Ove Ditlevsen. Aleatory or epistemic? does it matter? *Structural Safety*, 2009.
ISSN 0167-4730.
Risk Acceptance and Risk Communication

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- This division is purely subjective[2].
- For SPRA in the nuclear industry, epistemic uncertainties are the mechanical parameters of the structure (natural frequency, damping ratio,...) Aleatory uncertainties comes from the seismic ground motions' stochasticity.

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[2] Armen Der Kiureghian and Ove Ditlevsen. Aleatory or epistemic? does it matter?
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Fragility curve with epistemic uncertainties

Our goal is to assess the effect of mechanical model parameters uncertainties (e.g. epistemic uncertainties) on the QoI.

- For X a random vector of model parameters, $\Psi(a,X) = \mathbb{P}(Y > C|A = a,\ X)$
- The random function $\Psi(.,X)$ could be seen as a functional QoI.
- Scalar quantities derived from $\Psi(.,X)$ could also been used such as:

$$\Upsilon(X) = \int \Psi(a,X)dh(a)$$

Fragility curve estimation

- Fragility curves are estimated by Gaussian Process regression[3] :

$$\log(Y) = \beta_0 + \beta_1 \log(A) + Z(A, X) + \varepsilon \ ,$$

where Z a centered Gaussian Process with Matèrn 5/2 kernel and $\varepsilon \sim \mathcal{N}(0, \sigma^2)$.

- Kernel hyperparameters and σ are fitted by maximum likelihood.

- Given a dataset $\mathcal{D}_n = (A_i, X_i, Y_i)_{1 \leq i \leq n}$, the predictive distribution is $(\log(Y(a, X))|\mathcal{D}_n) \sim \mathcal{N}\left(\widehat{\log(Y)}(a, X),\ \widehat{\sigma}(a, X)^2\right)$.

$$\widehat{\Psi}(a, X) = \mathbb{P}(Y > C | A = a, X) = \Phi\left(\frac{\widehat{\log(Y)}(a, X) - \log(C)}{\widehat{\sigma}(a, X)}\right)$$

---

[3] Bertrand Iooss and Loïc Le Gratiet. Uncertainty and sensitivity analysis of functional risk curves based on gaussian processes. *Reliability Engineering & System Safety*, 187:58–66, 2019

Global Sensitivity Analysis framework

In order to study the influence of the uncertainties of the model parameters $X = (X^1, ..., X^p)$ on the scalar QoI $\Upsilon(X) = \int \Psi(a, X) dh(a)$, we recall the general formulation of sensitivity indices for parameter X^i [4]:

$$S_i = \mathbb{E}_{X^i}[d(\mathbb{P}_\Upsilon, \mathbb{P}_{\Upsilon|X^i})] \ ,$$

$d(.,.)$ is a dissimilarity between probability measures. Recall that Sobol indices are obtained with

$$d(\mathbb{P}_\Upsilon, \mathbb{P}_{\Upsilon|X^i}) = (\mathbb{E}[\Upsilon] - \mathbb{E}[\Upsilon|X^i])^2$$

Moreover, aggregated Sobol indices are defined naturally as:

$$S_i = \frac{\mathbb{E}[\|\bar{\Psi} - \Psi_{X^i}\|_{L^2}^2]}{\mathbb{E}[\|\bar{\Psi} - \Psi_X\|_{L^2}^2]} \ ,$$

with $\bar{\Psi} = \mathbb{E}[\Psi(., X)]$ and $\Psi_{X^i} = \mathbb{E}[\Psi(., X)|X^i]$.

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[4] Sebastien Da Veiga. Global sensitivity analysis with dependence measures. *Journal of Statistical Computation and Simulation*, 2015

MMD-based sensitivity indices

Recently the squared **Maximum Mean Discrepancy (MMD)** distance has been investigated [5]:

$$\mathrm{MMD}^2(\mathbb{P}_1, \mathbb{P}_2) = \mathbb{E}_{\xi,\xi' \sim \mathbb{P}_1}[k(\xi, \xi')] - 2\mathbb{E}_{\xi,\zeta \sim \mathbb{P}_1 \times \mathbb{P}_2}[k(\xi, \zeta)] + \mathbb{E}_{\zeta,\zeta' \sim \mathbb{P}_2}[k(\zeta, \zeta')]$$

Given a kernel $k(.,.)$, MMD based sensitivity indices are easily expressed using expectations:

$$S_i^{\mathrm{MMD}} = \mathbb{E}_{X^i}[\mathbb{E}_{Z,\tilde{Z} \sim \mathbb{P}_{\Upsilon|X^i}}[k(Z, \tilde{Z})]] - \mathbb{E}_{Z,\tilde{Z} \sim \mathbb{P}_{\Upsilon}}[k(Z, \tilde{Z})]$$

Pick freeze estimation of S_i^{MMD} and an ANOVA decomposition is proposed in the same reference.

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[5] Sébastien da Veiga. Kernel-based anova decomposition and shapley effects – application to global sensitivity analysis, 2021. arXiv: 2101.05487

Interpretation of the MMD sensitivity indices

We use a theoretical result on the MMD distance to raise an interpretation of the MMD based sensitivity indices [6]

Lemma

Given $k(x,y) = (\Phi * \Phi)(x-y)$ *with* $\Phi \in L^1(\mathbb{R})$. *Then,*

$$S_i^{MMD} = (2\pi)^{-1/4} \mathbb{E}_{X^i}\left[\|\Phi * \mathbb{P}_\Upsilon - \Phi * \mathbb{P}_{\Upsilon|X^i}\|^2_{L^2(\mathbb{R})} \right]$$

- Define $\Phi_\gamma(x) = \frac{1}{\gamma}\Phi\left(\frac{x}{\gamma}\right)$ with $\int \Phi(x)dx = 1$. Using the lemma above, we raise that:

$$S_i^{\text{MMD}} \xrightarrow[\gamma\to 0]{} (2\pi)^{-1/4} \mathbb{E}_{X^i}\left[\|p_\Upsilon - p_{\Upsilon|X^i}\|^2_{L^2(\mathbb{R})} \right]$$

where $d\mathbb{P}_\Upsilon = p_\Upsilon d\mu$ and $d\mathbb{P}_{\Upsilon|X^i} = p_{\Upsilon|X^i} d\mu$.

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[6] Bharath K. Sriperumbudur, Arthur Gretton, Kenji Fukumizu, Bernhard Schölkopf, and Gert R.G. Lanckriet. Hilbert space embeddings and metrics on probability measures.
Journal of Machine Learning Research, 2010

Empirical bandwidth parameter selection

- Considering an empirical measure $\widehat{\mathbb{P}}_{\Upsilon} = \frac{1}{n}\sum \delta_{\Upsilon_i}$:

$$(\Phi_\gamma * \widehat{\mathbb{P}}_{\Upsilon})(y) = \frac{1}{n\gamma}\sum_{i=1}^{n} \Phi\left(\frac{y - \Upsilon_i}{\gamma}\right)$$

- **Empirical MMD based sensitivity indices are the mean of the squared L^2 norm between kernel density estimators (KDE) estimators of $\mathbb{P}_{\Upsilon}$ and $\mathbb{P}_{\Upsilon|X^i}$.**

- We propose to choose a data-driven bandwidth $\widehat{\gamma}_n$ from the integrated MSE for KDE estimation:

$$\widehat{\gamma}_n = \underset{\gamma}{\operatorname{argmin}}\, \mathbb{E}[\|\Phi_\gamma * \widehat{\mathbb{P}}_{\Upsilon} - p_{\Upsilon}\|^2_{L^2(\mathbb{R})}]$$

Numerical application: Nonlinear oscillator

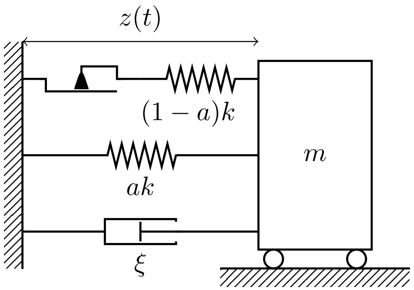

Figure: Elasto-plastic mechanical oscillator with kinematic hardening. The linear parameters are the mass m, stiffness k, damping ratio ξ. The non linearity is controlled by the yield limit Y and the post-yield stiffness a.

$$\ddot{z}(t) + 2\beta\omega_L\dot{z}(t) + f_{NL}(t) = -s(t) \ ,$$

Table: Epistemic uncertainties on the elasto-plastic oscillator

parameter	distribution	mean	c.o.v
m	Lognormal	1	10%
k	Lognormal	900	30%
ξ	Lognormal	0.02	50%
Y	Lognormal	5×10^{-3}	30%
a	Lognormal	0.2	30%

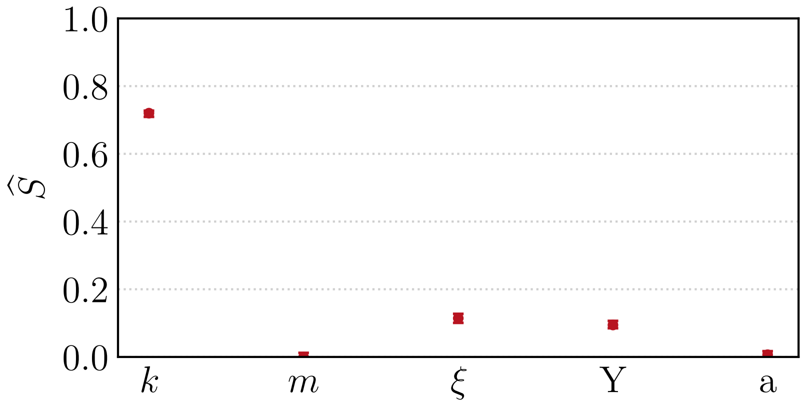

Figure: Aggregated first order Sobol indices on the fragility curve, 20 replications with sample size $n = 10,000$

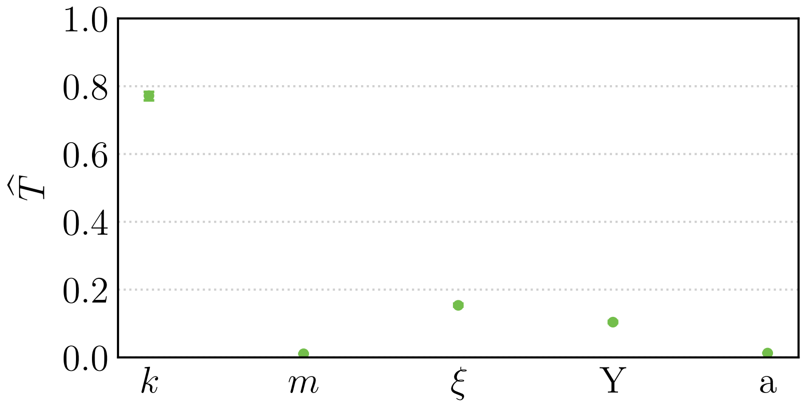

Figure: Aggregated total order Sobol indices on the fragility curve, 20 replications with sample size $n = 10,000$

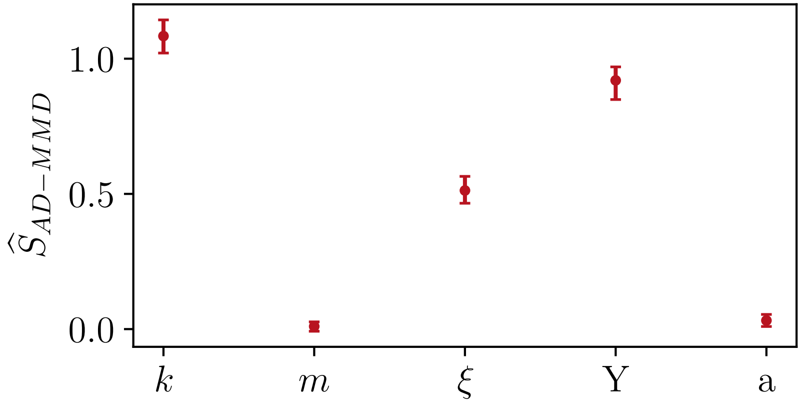

Figure: MMD based indices on the probability of failure Υ using Silverman's rule of thumb, 20 replications with sample size $n = 10,000$

Conclusion

- Definition of a metamodel on bouth epistemic and seismic uncertainties.
- Global Sensitivity Analysis framework on epistemic uncertainties on fragility curves' related QoIs.
- MMD-based sensitivity indices to handle more complex input/output relationships.

Thank you for your attention !

Références

Sébastien da Veiga. Kernel-based anova decomposition and shapley effects – application to global sensitivity analysis, 2021. arXiv: 2101.05487.

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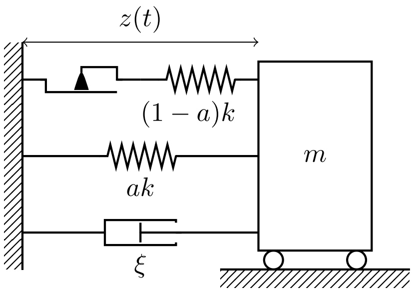
Appendix

Nonlinear oscillator



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Merci pour votre attention !

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