Active learning strategies for seismic fragility curve estimation with guarantees

Clément Gauchy (CEA/DES/ISAS Saclay)

Mail: *clement.gauchy@cea.fr*

Supervisors: Josselin Garnier (Ecole polytechnique), Cyril Feau (CEA)

1. Introduction

Seismic fragility curve is a key quantity of interest for seismic risk assessment. The binary random variable S gives the state of the system (0: idle, 1: failure) and the real r.v. $X \in \mathbb{R}$ corresponds to the logarithm of a seismic intensity measure IM (such as peak ground acceleration).

Observation of S requires expensive numerical simulations, conversely the observation of X is considered cheap. We suppose a binary regression model

$$S|X \sim \mathcal{B}(\mu(X)) \,,$$

where $\mu : \mathbb{R} \to [0,1]$ is the conditional mean of S, which defines the fragility curve.

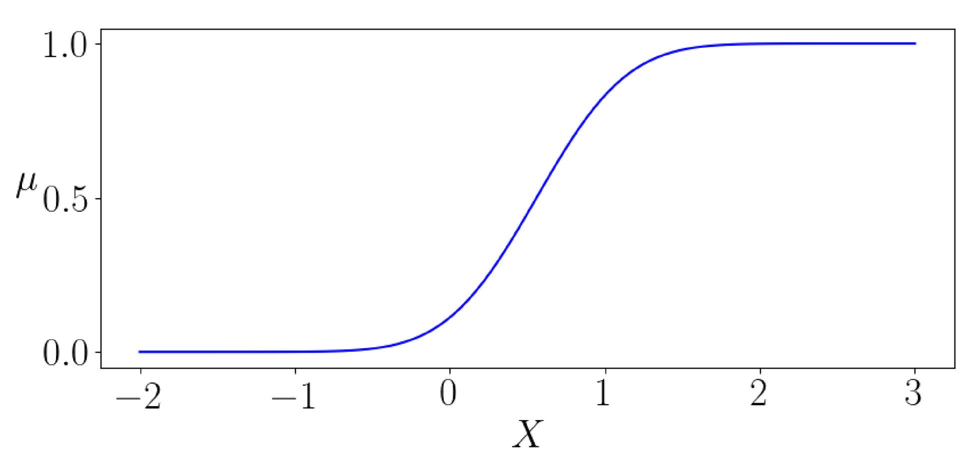

Figure 1: *Graphical representation of a fragility curve.*

2. Adaptive importance sampling

Given X with pdf p and a space of $[0,1]$-valued functions $\mathcal{F} = \{f_\theta, \theta \in \Theta \subset \mathbb{R}^m\}$, we want to estimate

$$\theta_0 = \underset{\theta \in \Theta}{\operatorname{argmin}}\, \mathbb{E}[\mu(X) - f_\theta(X))^2] = \underset{\theta \in \Theta}{\operatorname{argmin}}\, \mathbb{E}[\ell_\theta(X,S)]$$

with $\ell_\theta(x,s) = (s - f_\theta(x))^2$.

Considering X_i's iid with pdf p, the empirical estimator is

$$\widehat{\theta}_n = \underset{\theta \in \Theta}{\operatorname{argmin}}\, \frac{1}{n}\sum_{i=1}^{n} \ell_\theta(X_i, S_i) \,.$$

Importance sampling [1] may be used to reduce training loss variance. The distribution of the X_i's is replaced by a well-chosen instrumental distribution that minimizes asymptotic variance:

$$q_\theta(x) \propto \sqrt{\mathbb{E}[\ell_\theta(X,S)^2 | X = x]}\, p(x) \,.$$

⚠ The optimal distribution q_θ^* may increase dramatically the variance for small n [1], so we use a defensive instrumental distribution:

$$q_{\theta,\varepsilon}(x) = \varepsilon p(x) + (1-\varepsilon) q_\theta(x), \quad \varepsilon \in (0,1),$$

The instrumental distribution depending on θ, the algorithm is called adaptive importance sampling (AIS) [2], the AIS estimator $\tilde{\theta}_n$ is defined as follows:

- (1) Initialize θ_0 ,
- (2) $\tilde{R}_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} \frac{p(X_i)}{q_{\tilde{\theta}_{i-1},\varepsilon}(X_i)} \ell_\theta(X_i, S_i)$,
- (3) $\tilde{\theta}_n = \operatorname{argmin} \tilde{R}_n(\theta)$.

AIM OF THE PhD

The purpose of this PhD is to alleviate computational burden for seismic fragility curve estimation and to propagate epistemic uncertainties from the mechanical model to seismic fragility curves. An active learning algorithm is used to reduce training loss variance and accelerate convergence. Epistemic uncertainties propagation will be studied in further works.

3. Theoretical results

The main theoretical result is the asymptotic normality of the AIS estimator $\tilde{\theta}_n$

Theorem: Denote $r(\theta) = \mathbb{E}[\ell_\theta(X,S)]$. With appropriate regularity conditions, $\sqrt{n}(\tilde{\theta}_n - \theta_0)$ is asymptotically normal with mean zero and covariance matrix $\ddot{r}(\theta_0)^{-1} V(q_{\theta_0,\varepsilon}, \ell_{\theta_0})(\ddot{r}(\theta_0)^{-1})^T$ with

$$V(q_{\theta_0,\varepsilon}, \ell_{\theta_0}) = \mathbb{E}\left[\frac{p(X)\nabla\ell_{\theta_0}(X,S)\nabla\ell_{\theta_0}(X,S)^T}{q_{\theta_0,\varepsilon}(X)}\right]$$

The proof arguments are the same as for classical M-estimation [3], we only have to consider that $\tilde{R}_n(\theta)$ is a martingale.

4. Numerical results

Introduced in the early 1980s for nuclear safety assessment, the lognormal model $\mathcal{F} = \{\Phi(\frac{\log(\frac{IM}{\alpha})}{\beta}), (\alpha,\beta) \in \mathbb{R}^{+*} \times \mathbb{R}^{+*}\}$, where Φ is the cdf of the standard Gaussian distribution and $X = \log(IM)$, is widely used in many applications.

The studied model is a nonlinear mechanical oscillator with kinematic hardening [4] subjected to artificial seismic signals, the seismic intensity measure considered in this case is the peak ground acceleration (PGA). The study is done with a pool of 20,000 unlabeled datapoints.

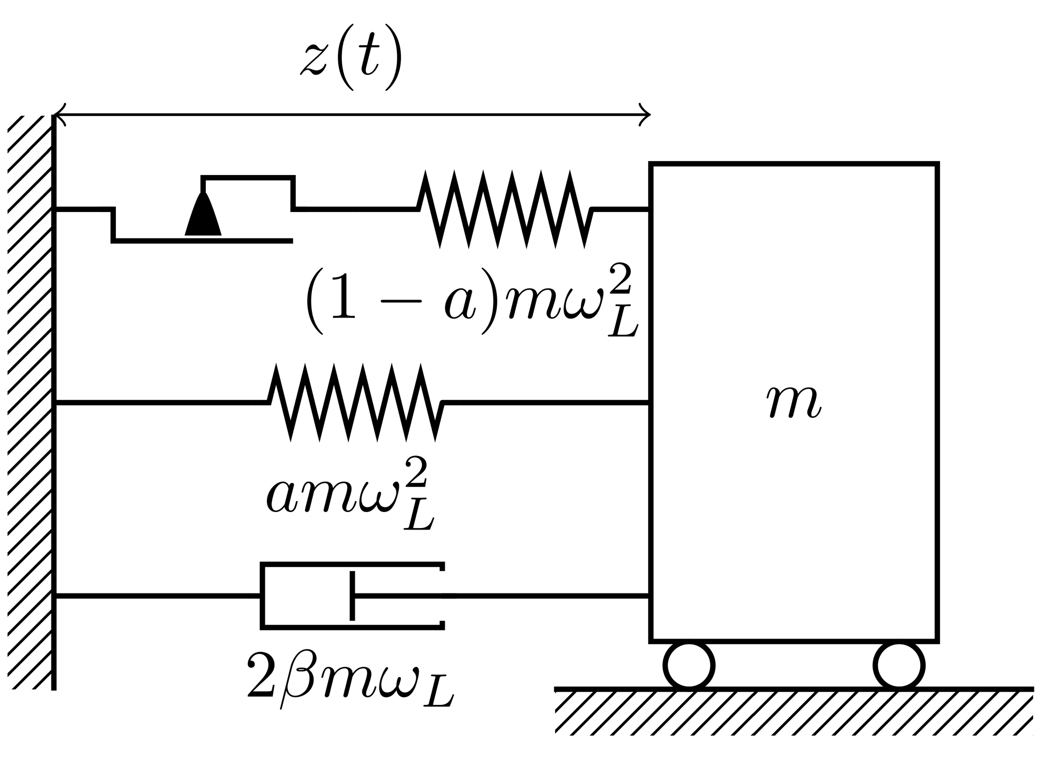

Figure 2: *Nonlinear mechanical oscillator with kinematic hardening with parameter, $f_L = 5$ Hz, $\beta = 2\%$, the yield limit is $Y = 5.10^{-3}$ m and the post-yield stiffness is 20% of the elastic stiffness hence $a = 0.2$*

Setting $Z = \max_{t \in [0,T]} |z(t)|$. The failure for this system is for instance defined by

$$S = \mathbb{1}_{(Z > Y)}$$

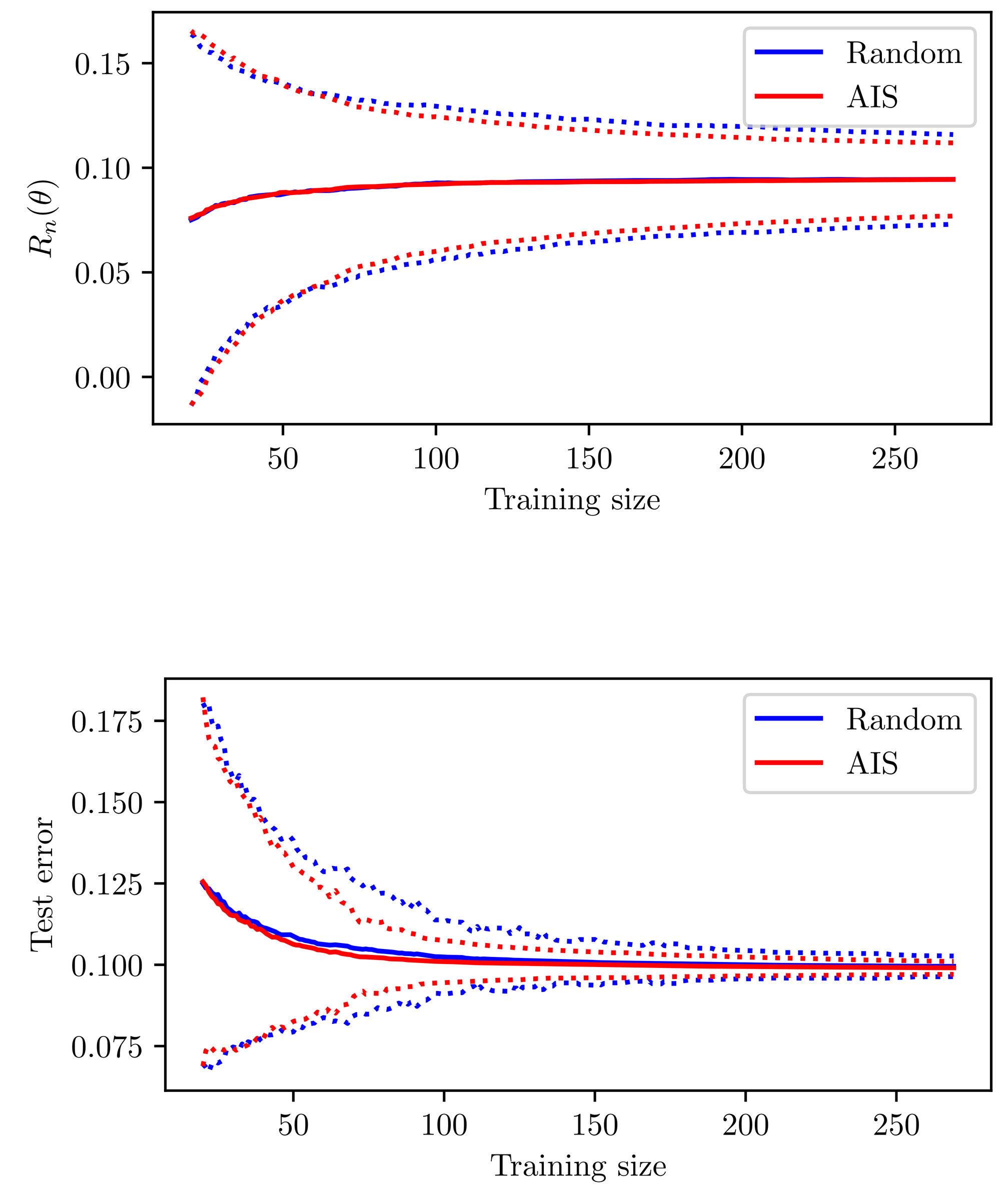

Figure 4: *Mean train error for random strategy and defensive AIS with a mixing parameter $\varepsilon = 10^{-2}$. Initial size of 20 random observations, with 500 replications of 250 labeled datapoints using AIS. The dotted lines represent the confidence intervals ($\pm 2\sigma$). The training and testing error's variances are reduced thanks to AIS.*

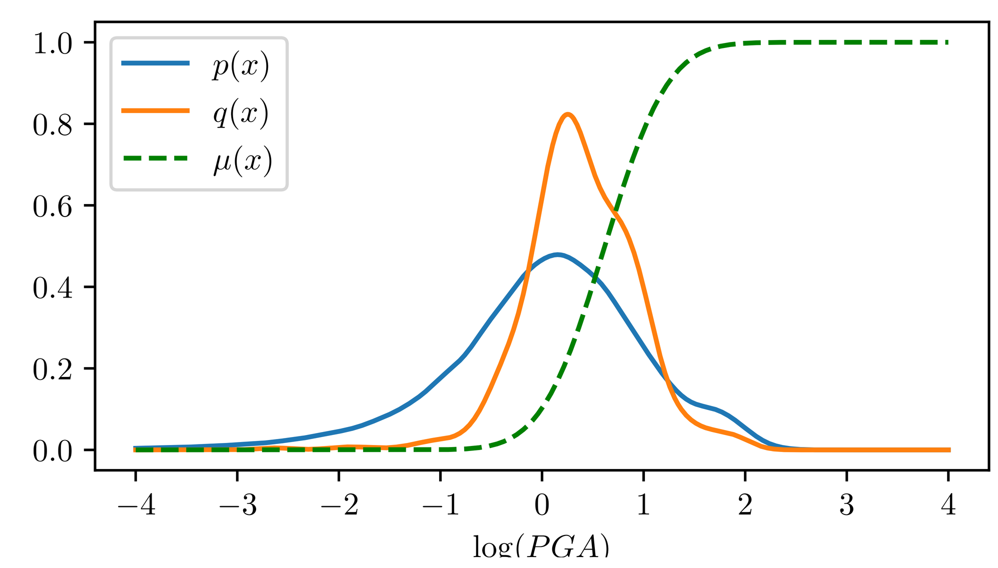

Figure 5: *Graphical representation of the lognormal fragility curve estimate with 20,000 datapoints, the original distribution $p(x)$ and the defensive instrumental distribution $q(x)$ with $\varepsilon = 10^{-2}$.*

As illustrated in Figure 5, AIS queries intensity measures in the transition region of the fragility curve between no failure and certain failure of the system. This behaviour decreases the estimator's variance.

CONCLUSION

Having both motivating numerical results and theoretical guarantees, AIS is a well suited active learning strategy. A remaining issue is to improve initialization of the algorithm.

References

[1] A. Owen and Y. Zhou. Safe and effective importance sampling, *Journal of the American Statistical Association*, March 2000.

[2] B. Deylon and F. Portier. Asymptotic optimality of adaptive importance sampling, *NIPS 2018*.

[3] A. W. van der Vaart., Asymptotic statistics, *Cambridge University Press*, 1998.

[4] R. Saint, C. Feau, J.-M. Martinez, J. Garnier, Efficient methodology for seismic fragility curves estimation by active learning on Support Vector Machines, *Structural Safety*, September 2020.